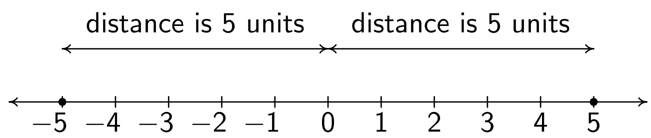
MATH 1650: SECTION A.7: ABSOLUTE VALUE REVIEW

DISTANCE DEFINITION OF ABSOLUTE VALUE: The **absolute value** of a real number x, denoted $|x|$, is the distance between x and 0 on the number line.

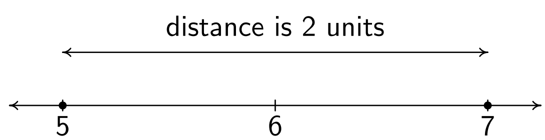
For example, $|5| = 5$ and $|-5| = 5$, since both 5 and -5 are 5 units from 0 on the number line:



Graphically why $|-5| = 5$ and $|5| = 5$

More generally, $|x - c|$ is the distance between the numbers x and c on the number line.

For example, $|5 - 7| = |-2| = 2$ since 5 and 7 are two units away from each other on the number line:



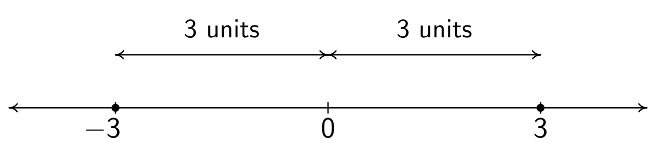
Graphically why $|5 - 7| = 2$

PROPERTIES OF ABSOLUTE VALUE:

- **Product Rule:** $|ab| = |a||b|$
- **Power Rule:** $|a^n| = |a|^n$
- **Quotient Rule:** $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$, provided $b \neq 0$
- **TRIANGLE INEQUALITY:** $|a + b| \leq |a| + |b|$.

  NOTE: In general, $|a + b| \neq |a| + |b|$.

EXAMPLE: Solve $|x| = 3$. Thinking in terms of distance, we are looking for all real numbers x whose distance from 0 is 3 units. If we move three units to the right of 0, we end up at $x = 3$. If we move three units to the left, we end up at $x = -3$. Thus the solutions to $|x| = 3$ are $x = \pm 3$.



The solutions to $|x| = 3$ are $x = \pm 3$.

IN GENERAL: If $c \geq 0$, the solutions to $|x| = c$ are $x = \pm c$.

NOTE: If $c < 0$, $|x| = c$ has no solution since $|x|$ is never negative. For example $|x| = -1$ has no solution.

STRATEGY FOR SOLVING EQUATIONS INVOLVING ABSOLUTE VALUE:

In order to solve an equation involving the absolute value of a quantity $|X|$:

1. Isolate the absolute value on one side of the equation so it has the form $|X| = c$.
2. As long as $c \geq 0$, solve the equations: $X = c$ or $X = -c$.

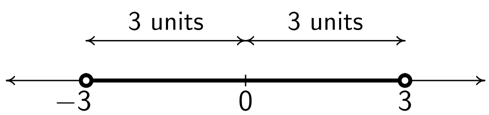
   NOTE: If $c = 0$, we get just one equation, $X = 0$. If $c < 0$, there is no solution.

EXAMPLE: Solve each of the following equations.

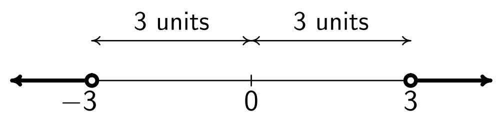
1. $|3x - 1| = 6$
2. $\dfrac{3 - |y + 5|}{2} = 1$
3. $3|2t + 1| - 3 = 0$

EXAMPLE:

- Solve $|x| < 3$. Geometrically, we are looking for all of the real numbers whose distance from 0 is *less* than 3 units. We get $-3 < x < 3$, or in interval notation, $(-3, 3)$.
- Solve $|x| > 3$. Now we want the distance between x and 0 to be *greater* than 3 units. Moving in the positive direction, this means $x > 3$. In the negative direction, this puts $x < -3$. Our solutions would then satisfy $x < -3$ *or* $x > 3$. In interval notation, we express this as $(-\infty, -3) \cup (3, \infty)$.



The solution to $|x| < 3$ is $(-3, 3)$



The solution to $|x| > 3$ is $(-\infty, -3) \cup (3, \infty)$

Generalizing this notion, we get the following:

IN GENERAL: For real numbers $c > 0$:

- $|x| < c$ is equivalent to the compound inequality: $-c < x < c$.
- $|x| > c$ is equivalent to the compound inequality: $x < -c$ or $x > c$.

STRATEGY FOR SOLVING INEQUALITIES INVOLVING THE ABSOLUTE VALUE:

In order to solve an inequality involving the absolute value of a quantity $|X|$:

1. Isolate the absolute value on one side of the inequality to get in the form $|X| < c$ or $|X| > c$.
2. Assuming $c > 0$:
   - rewrite $|X| < c$ as the compound inequality: $-c < X < c$;
   - rewrite $|X| > c$ as the compound inequality: $X < -c$ or $X > c$.

   NOTE: If $c \leq 0$, the solution set could be all real numbers, a single real number, or have no solution.

EXAMPLE: Solve the following inequalities. Write your answer using interval notation.

1. $|x - 5| > 1$

2. $\dfrac{4 - 2|2x + 1|}{4} \geq -3$

ADDITIONAL REVIEW: Section A.7: 1 - 17 odd.